Correspondence

The Negative Capacitor, An Impedance Matching Element for Dielectric-Filled Transmission Line

The well-known problem of a capacitive dielectric window in an air-filled cylindrical transmission line[1] can be extended to the general problem in which the transmission line is dielectric filled, as shown in Fig. 1. This leads to a unique result. When the dielectric constant of the window is less than that of the loading material, the window can be represented by a shunt negative capacitor at the reference plane Z_0 in the uniformly filled line. This result does not contradict Foster's reactance theorem.[2] The negative capacitor is a lumped parameter representation of a distributed-parameter network and, as such, has no accessible port. The lumped-parameter representation–even though it entails some approximations–facilitates the use of the window for impedance matching.

The shunt capacitance equivalent circuit of the window is shown in Fig. 2. Ports a and b correspond to the points $Z_0-\frac{1}{2}d$ and $Z_0+\frac{1}{2}d$, respectively. Two lengths of transmission line having the propagation characteristics of the loaded line are included in the equivalent circuit. By constraining these to length $\frac{1}{2}d$, the window is represented as a shunt capacitor at Z_0. This constraint introduces an approximation into the circuit. The degree of approximation can be determined by comparison with the exact equivalent circuit, shown in Fig. 3, a length of transmission line joining ports a and b. Its impedance transfer matrix (T_1) is given by

$$(T_1) = \begin{pmatrix} \cos \kappa_2 d & jZ_2 \sin \kappa_2 d \\ jY_2 \sin \kappa_2 d & \cos \kappa_2 d \end{pmatrix}. \qquad (1)$$

The characteristic impedance, propagation constant, and guide wavelength in the window region are denoted as Z_2, κ_2, and λ_2, respectively. The corresponding quantities for the uniformly filled transmission line have the subscript 1. The impedance transfer matrix (T_2) for the lumped-capacitor equivalent circuit is given by the following:

$$(T_2) = \begin{pmatrix} \cos \frac{1}{2}\kappa_1 d & jZ_1 \sin \frac{1}{2}\kappa_1 d \\ jY_1 \sin \frac{1}{2}\kappa_1 d & \cos \frac{1}{2}\kappa_1 d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ jB & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos \frac{1}{2}\kappa_1 d & jZ_1 \sin \frac{1}{2}\kappa_1 d \\ jY_1 \sin \frac{1}{2}\kappa_1 d & \cos \frac{1}{2}\kappa_1 d \end{pmatrix} \qquad (2)$$

The lumped parameter equivalent circuit will be valid to the degree that (T_2) approximates (T_1).

Considering only TE modes (including the limiting case of the TEM mode), setting the right-hand side of (2) equal to the right-hand side of (1) leads to the following result.

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[1] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, *Principles of Microwave Circuits*, Rad. Lab. Ser., vol. 8 New York: McGraw-Hill, 1948, pp. 374–375.

[2] R. M. Foster, "A reactance theorem," *Bell Sys. Tech. J.*, vol. 3, no. 2, pp. 259–260, 1923.

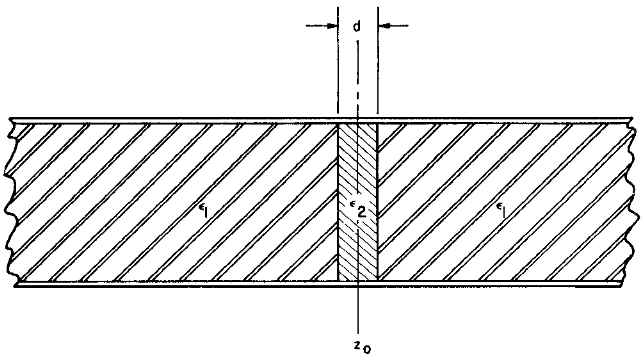

Fig. 1. Dielectric window in uniformly loaded transmission line.

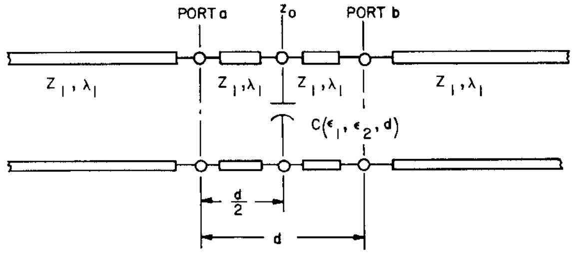

Fig. 2. Lumped-element representation of window.

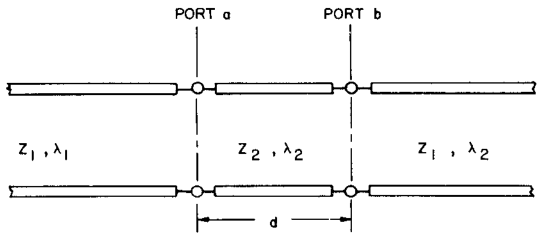

Fig. 3. Exact transmission-line circuit.

If the length of the window is small enough that $\tan \frac{1}{2}\kappa_1 d$ and $\tan \kappa_2 d$ can be approximated by $(\frac{1}{2})\kappa_1 d$ and $\kappa_2 d$, respectively, the shunt capacitance representation may be used. It should be noted that it makes no difference if κ_2 is real or imaginary. The resultant formula for the normalized susceptance is

$$\frac{B}{Y_1} = \frac{2\pi d\lambda_1}{\lambda^2}(\epsilon_2 - \epsilon_1). \qquad (3)$$

Multiplying both sides by Y_1, the wave admittance for a TE mode, gives the absolute susceptance

$$B = 2\pi f d(\epsilon_2 - \epsilon_1)\epsilon_0. \qquad (4)$$

This is the susceptance of a capacitor whose value is given by

$$C = (\epsilon_2 - \epsilon_1)\epsilon_0 d. \qquad (5)$$

When ϵ_1 is greater than ϵ_2, the window is a negative capacitor. This does not violate Foster's reactance theorem because the negative capacitor is not at an accessible port. The total equivalent circuit, which includes the two lengths of line, has terminal properties that obey Foster's theorem. In fact, the negative capacitor representation may be arrived at intuitively by considering the dielectric window solely from the point of view of the reactance theorem. Lowering the dielectric constant along the length of the window reduces the capacitance per unit length, resulting in a reduction of energy storage (or delay) between port a and port b. Since, however, the equivalent circuit includes a total length of line with the same propagation constant as the uniformly filled waveguide, the reduction in energy storage or "negative energy storage" must be accounted for by the shunt susceptance. From the original statement of Foster's theorem,[2] an element having "negative energy storage" will also have a negative susceptance slope (or reactance slope).

In addition to its use as an impedance matching element, the negative capacitor may be used to generate a susceptance, again at the point Z_0, which remains relatively constant over a broad frequency band. This problem has not been analyzed rigorously. However, an approximate formula was gen-

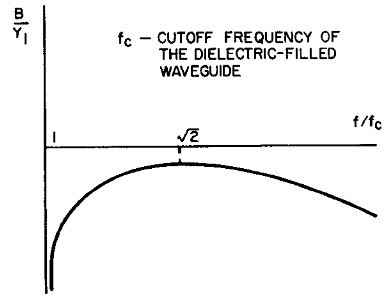

Fig. 4. Normalized susceptance for a negative capacitor in uniformly loaded waveguide.

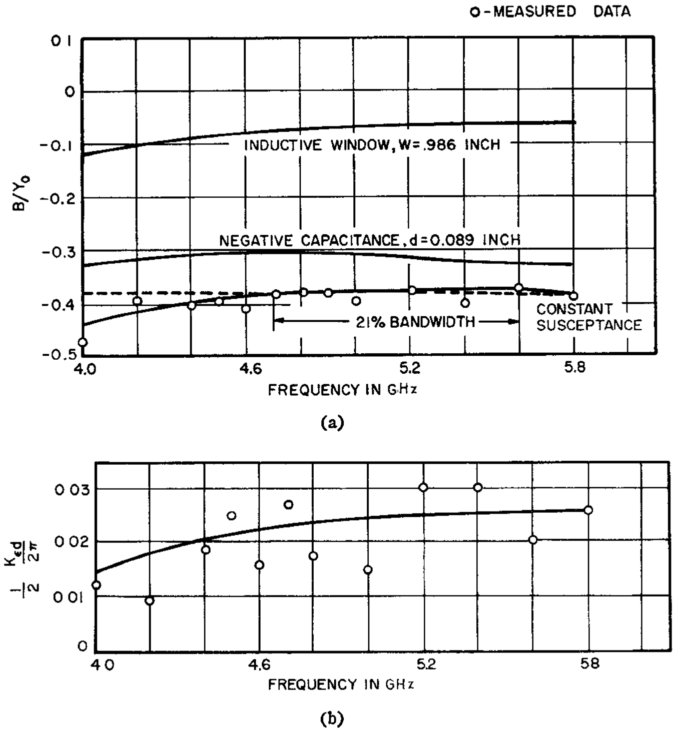

Fig. 5. Equivalent circuit parameters for experimental constant susceptance network. (a) Normalized susceptance. (b) Normalized line length.

erated, and relatively good experimental agreement obtained. Fig. 4 is a plot of the normalized susceptance of the negative capacitance in waveguide. The susceptance of an inductive iris has the same algebraic sign as that of the negative capacitor, but at frequencies greater than $\sqrt{2}f_c$ their frequency trends are opposite. When the slopes of the two elements are equal in magnitude, the magnitude of the inductive susceptance will be about one-fourth the magnitude of the negative-capacitive susceptance. Consequently, it seems reasonable that the introduction of a small inductive perturbation of proper magnitude into the window would produce a constant susceptance. Moreover, it is assumed that the total susceptance can be obtained by determining the susceptance of the inductive iris in the absence of the window, and adding this to the susceptance of the negative capacitance. This is particularly well founded since the iris perturbs the modal H field, which is not affected by the presence of the window, and the window provides a discontinuity to the modal E field, which is not significantly affected by the iris. Consequently using Marcuvitz's formula for the susceptance of the iris,[3] the formula for the total normalized susceptance is obtained

$$\frac{B_t}{Y_1} = -\frac{\lambda_1}{a}\cot^2\frac{\pi w}{2a} - \frac{2\pi d\lambda_1}{\lambda^2}(\epsilon_1 - \epsilon_2). \quad (6)$$

The validity of (6) was experimentally determined by measuring the impedance of a constant susceptance network in polystyrene ($\epsilon_1 = 2.56$) loaded WR112 waveguide. The length of the window was 0.089 inch, while ϵ_2 was unity. The iris aperture w was 0.986 inch. These values were determined by equating the values of (6) for 4.7 and 5.8 GHz. The nominal susceptance, over the 21 percent band, for these parameters is 0.375. Fig. 5 shows curves of the computed circuit parameters and measured data.

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[3] N. Marcuvitz, *Waveguide Handbook*, Rad. Lab. Ser., vol. 10. New York: McGraw-Hill, 1950, p. 221.

High-Power *S*-Band Power Divider

The problem of covering the full power range from milliwatts to megawatts in laboratory experiments, with a minimum number of RF power source, and instrumentation changes, has always existed in the field of microwaves. A high-power attenuator, covering a 20 or 30 dB power range using a single RF power source, is needed so that experiments can yield more complete data. The power source used could be set to a fixed frequency and output power level, and the attenuator could be adjusted to supply discrete amounts of power, as needed, to test an experimental device. The frequency of the power source would not have to be readjusted each time the power out of the attenuator is changed.

Since an in-line attenuator must absorb all of the energy it does not pass, it is automatically power limited. However, a power divider diverts the unused portion of the RF energy to an external dummy load. Since the power absorbed by the power divider itself is negligible, the only design problems are those of high-voltage breakdown and impedance matching.

The power divider discussed here provides a 30 dB dynamic range of power control and a peak power handling capability of at least 700 kilowatts at S band. A high peak power rating is necessary because there are some pulsed magnetrons that must operate at a minimum of 300 to 400 kilowatts in order to produce a stable output with a satisfactory pulse shape. A magnetron usually provides satisfactory operation over a dynamic power range of only 6 to 8 dB

The power divider consists basically of four units: 1) an input–output adapter section, 2) a 3 dB hybrid coupler, 3) a dual waveguide shutter assembly, and 4) a sidewall "panty" adapter (so called because of its unique shape). A pair of matched loads are also used, but they are external to the power divider. A block diagram of the power divider is shown in Fig. 1.

The input RF power from a magnetron or other high-power source is fed through the input arm of the input–output adapter section and into the 3 dB hybrid coupler. The RF input power, fed into port 1, splits into two equal amounts of power that pass out of ports 2 and 3, respectively. These two RF signals, which we shall designate as signals A and B, pass through the panty adapter and enter the RF dual shutter assembly.

The RF dual shutter assembly is constructed so that both halves of the shutter open and close in unison. If the shutters are both completely open, then all of signal A is absorbed as signal $A1$ by matched load A. Likewise, all of signal B is absorbed as signal $B1$ by matched load B. When the shutters are completely closed, then all of signals A and B are reflected as $A2$ and $B2$. Whenever the shutters are partially open, equal amounts of signals A and B are reflected.

The output power at port 4 will be equal to the sum of signals $A2$ and $B2$. Signal $A2$ leads signal $B2$ by 90 degrees at the input to the hybrid coupler, but signal $B2$ will receive a 90 degree phase lead going through the hybrid coupler from port 3 to port 4. As a

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